SEMI-CLASSICAL DESCRIPTION OF NUCLEAR DEFORMATIONS FROM SADDLE TO SCISSION

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Abstract

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The energy density formalism based on the Skyrme force has been used for calculating the deformation energies on the fission of ^{240}Pu. The kinetic energy density is expressed as a functional of the matter density and its derivatives. The fission shapes are described by two ellipsoids connected by a six-order polynomial. The influence of shape parameterization is discussed. Results are compared to both liquid-drop and self-consistent (Hartree-Fock) calculations. In agreement with liquid-drop calculations a well defined 'exit region' is found. Advantages of further developments of semi-classical approximations are discussed.

1. INTRODUCTION

A common feature of the majority of theoretical descriptions of strongly deformed nuclei is that they are based on the Strutinsky prescription. It is then assumed that the binding energy of the nucleus can be divided into two parts, namely one slowly varying part, which describes the average variation of the energy with respect to deformation and particle number and one strongly oscillating part, the so-called shell correction energy. The later part can be calculated, using a single-particle potential of e.g. Woods-Saxon type, the parameters of which are fitted in such a way that the single-particle levels in the ground state are well reproduced. However the total nuclear energy cannot be described by simply adding up the energies of the occupied states. Therefore the slowly varying part of the energy is replaced by a semiempirical energy, calculated for example by means of the liquid drop model. When extending these kinds of calculations to deformations involved in the fission process between the saddle point and the scission point it is obvious that the fall-off of the potential energy, which is of the order tens of MeV, is essentially determined by the liquid drop energy. Since in the later stages of this process a strong necking is developed, terms like curvature energy might be important, and therefore a refined version of the liquid drop model has to be used.

In this paper we shall discuss an alternative approach, which avoids the use of an energy of liquid drop type. In the last ten years considerable progress has been made in the application of Hartree-Fock calculations to heavy nuclei, using an effective interaction of Skyrme type. These kind of calculations reproduce quite well the ground state properties, as reported in ref. [1]. They also allow for a qualitative description of the fission barrier. However, at large deformations the computational difficulties become very serious and thus it is practically impossible to perform the calculations with high accuracy. There exists, however, a way to bypass the time-consuming evaluation of the Hartree-Fock equations, especially, if we are interested only in the averaged, smoothly varying quantities, since the use of semiclassical techniques allows us to find such a solution, without solving the quantum mechanical equations.

2. THE SEMI-CLASSICAL TECHNIQUE

An essential point in the semiclassical calculations is to make an expansion of the density matrix for a given Hamiltonian [2]. In order to obtain such an expansion one may introduce as in ref. [3] the Wigner transform, defined as

$$A_w(\bar{r},\bar{p}) = \int A\left(\bar{r}-\frac{\bar{r}''}{2},\ \bar{r}+\frac{\bar{r}''}{2}\right) e^{i\bar{p}\cdot\bar{r}''/\hbar}\, d^3r'' \tag{1}$$

where $A(\bar{r},\bar{r}') = \langle\bar{r}|\hat{A}|\bar{r}'\rangle$ are the matrix elements of the operator $\hat{A}$ in configuration space, and $(\bar{r},\bar{p})$ denotes a point in the classical phase space.

The Wigner transform can in a natural way be expanded in powers of $\hbar$:

$$A_w(\bar{r},\bar{p}) = \sum_{n=0}^{\infty} A_n(\bar{r},\bar{p})\,\hbar^n \tag{2}$$

which we call the semiclassical expansion of A. The lowest-order term, $A_0(\bar{r},\bar{p})$, corresponds to the classical limit of the operator $\hat{A}$.

The Wigner transform of the Hamiltonian, H_W, is independent of $\hbar$, i.e. H_W coincides with the classical Hamiltonian[3]. Then the Wigner transform of the density matrix can be written

$$\rho_w(\bar{r},\bar{p}) = \sum_{n=0}^{\infty} \rho_{2n}(\bar{r},\bar{p})\,\hbar^{2n} \tag{3}$$

and contains only even powers of $\hbar$ [3]. Once $\rho_W(\bar{r},\bar{p})$ is known it is an easy task to calculate various physical quantities. Thus the normal density is given by

$$\rho(\bar{r}) = \int \frac{d^3p}{(2\pi\hbar)^3}\cdot \rho_w(\bar{r},\bar{p}) \tag{4}$$

and the kinetic energy distribution by

$$\tau(\bar{r}) = \frac{1}{4}\nabla^2\rho(\bar{r}) + \int \frac{d^3p}{(2\pi\hbar)^3}\cdot\frac{p^2}{\hbar^2}\cdot\rho_w(\bar{r},\bar{p}) \tag{5}$$

For nucleons interacting with a Skyrme-force the Hamiltonian in Wigner-space can be written

$$H_w = \frac{1}{2m}f(\bar{r})\,p^2 + V(\bar{r}) \tag{6}$$

where $f(\bar{r})$ is the effective mass and $V(\bar{r})$ is the potential. Since H_W is independent of $\hbar$, the expansion of $\rho_W(\bar{r},\bar{p})$ is given by (3), and one finds, using the eqs. (4) and (5) that

$$\rho = \frac{1}{3\pi^2} k_F^3 + \mathcal{O}(\hbar^2) \tag{7}$$

$$\tau = \frac{1}{5\pi^2} k_F^5 + \mathcal{O}(\hbar^2) \tag{8}$$

The local Fermi momentum $p_F = k_F \hbar$ is given by

$$\frac{1}{2m} f p_F^2 + V = E_F \tag{9}$$

where E_F is the Fermi energy. The zeroth order terms are immediately recognized as the Thomas-Fermi expressions.

Both $\rho(\bar{r})$ and $\tau(\bar{r})$ are functionals of the potential $V(\bar{r})$ and the Fermi energy E_F. It is however possible to eliminate the dependence on $V(\bar{r})$ and E_F, leading to the following expansion for τ:

$$\tau[\rho] = \tau_0[\rho] + \tau_2[\rho] + \tau_4[\rho] + \ldots \tag{10}$$

Thus τ is a functional of ρ, and a given term τ_n is originating from the terms of order $\hbar^n$ in the Wigner-expansion. The lowest order term is given by

$$\tau_0[\rho] = \frac{3}{5}(3\pi^2)^{2/3} \rho^{5/3} \tag{11}$$

which is the usual Thomas-Fermi term. The expressions for τ_2 and τ_4 can be found in refs [4,3,11].

It should be pointed out that the semiclassical expansions (7) and (8) are only valid up to the classical turning-point, i.e. up to $p_F(\bar{r}) = 0$. Thus the functional $\tau[\rho]$ is in principle not defined outside this point. Although there exist methods for overcoming this limitation [5], we shall not use them here, since it has been shown [6] that assuming the validity of the functional (10) also outside the classical turning point leads to correct results. However, the density given by eq. (7) is in any case unrealistic, since it is undefined outside the turning point. We shall therefore not try to calculate $\rho(\bar{r})$ from the Skyrme-potential by means of eq. (7), but make an ansatz for $\rho(\bar{r})$, which assures a realistic fall off of the density outside the classical turningpoint (see next section). We will then not get a fully selfconsistent density. Instead we optimize the parameterization of the density by minimizing the total energy with respect to the parameters, ρ_0, describing the central density and a_s, describing the diffuseness of the surface. However, for a first application of the semi-classical technique to the fission process we find this simplification justified. The calculation of the total energy E, then becomes simple and can be summarized in the following formulas:

$$E = \int e(\rho)\, d\Omega \tag{12}$$

$$e(\rho) = \frac{\hbar^2}{2m} \cdot \tau(\bar{r}) + \mathcal{V}(\bar{r}) + C(\bar{r}) \tag{13}$$

The Skyrme energy density $\mathcal{V}(\vec{r})$ [7] is given by

$$\begin{aligned}\mathcal{V}(\vec{r}) &= \tfrac{1}{2}t_0\left[(1+\tfrac{1}{2}X_0)\rho^2 - (X_0+\tfrac{1}{2})(\rho_n^2+\rho_p^2)\right] + \tfrac{1}{4}t_3\rho\rho_n\rho_p \\ &+ \tfrac{1}{16}(t_2-3t_1)\rho\Delta\rho + \tfrac{1}{32}(3t_1+t_2)(\rho_n\Delta\rho_n + \rho_p\Delta\rho_p) \\ &- \tfrac{1}{2}W_0(\rho\vec{\nabla}\cdot\vec{J} + \rho_n\vec{\nabla}\cdot\vec{J}_n + \rho_p\vec{\nabla}\cdot\vec{J}_p) + \tfrac{1}{4}(t_1+t_2)\rho\tau + \tfrac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p)\end{aligned} \tag{14}$$

where $\rho = \rho_n + \rho_p$, $\tau = \tau_n + \tau_p$ and the spin-density $\vec{J} = \vec{J}_n + \vec{J}_p$, for which the semiclassical expression (given for the neutrons) is [11]

$$\vec{J}_n = -\frac{2m}{\hbar^2}\,\vec{W}_n\,\frac{\rho_n}{f_n} \tag{15}$$

with

$$\vec{W}_n = W_0\left[\vec{\nabla}\rho_n + \tfrac{1}{2}\vec{\nabla}\rho_p\right] \tag{16}$$

The effective mass is

$$f_n = 1 + \frac{2m}{\hbar^2}\left(\alpha\rho_n + \beta\rho_p\right) \tag{17}$$

with

$$\alpha = (t_1 + 3t_2)/8 \tag{18}$$

and

$$\beta = (t_1 + t_2)/4 \tag{19}$$

Integrating all the τ-dependent terms of eq. (13) gives

$$T = T_0 + T_2 + T_4 \tag{20}$$

where

$$T_0 = \frac{\hbar^2}{2m}\cdot\frac{3}{5}(3\pi^2)^{2/3}\int d\Omega\left[\rho_n^{5/3} + \rho_p^{5/3} + \alpha\left(\rho_n^{8/3}+\rho_p^{8/3}\right) + \beta\left(\rho_p\rho_n^{5/3} + \rho_n\rho_p^{5/3}\right)\right] \tag{21}$$

$$T_2 = \frac{\hbar^2}{2m}\int d\Omega\left\{\frac{1}{36}\left(\frac{|\nabla\rho_n|^2}{\rho_n} + \frac{|\nabla\rho_p|^2}{\rho_p}\right) - \frac{11}{36}\left[\alpha\left(|\nabla\rho_n|^2 + |\nabla\rho_p|^2\right) + 2\beta\nabla\rho_n\cdot\nabla\rho_p\right]\right\} - \tfrac{1}{2}E_{so} \tag{22}$$

(E_{so} denotes the spin-orbit term of eq. (14))

$$\begin{aligned}\text{and } T_{4\nu} = \frac{\hbar^2}{2m}(3\pi^2)^{-2/3}\int d\Omega\Bigg\{&\rho_\nu^{1/3}\left[\frac{1}{810}\left|\frac{\nabla\rho_\nu}{\rho_\nu}\right|^4 - \frac{1}{240}\,\frac{|\nabla\rho_\nu|^2\Delta\rho_\nu}{\rho_\nu^3} + \frac{1}{270}\left(\frac{\Delta\rho_\nu}{\rho_\nu}\right)^2\right] \\ &+ \rho_\nu^{-2/3}\left[\frac{1}{108}(\Delta\rho_\nu)^2 - \frac{1}{1080}\,\frac{|\nabla\rho_\nu|^2\Delta\rho_\nu}{\rho_\nu} - \frac{7}{3240}\,\frac{|\nabla\rho_\nu|^4}{\rho_\nu^2}\right]\Bigg\}\end{aligned} \tag{23}$$

+ terms depending on α, β and $\vec{J}$; $\nu = n, p$

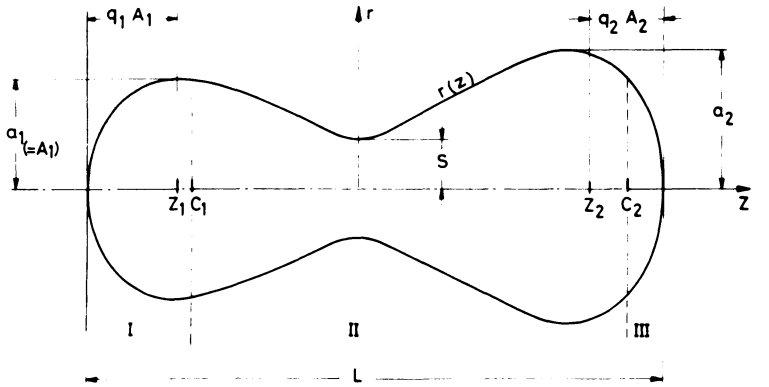

FIG.1. Geometrical meaning of some of the deformation parameters. Left-hand side: matching point (c_1) between ellipsoid centre and neck; right-hand side: matching point (c_2) close to end of nucleus.

We do not calculate the Coulomb energy by integrating the Coulomb energy density $C(\bar{r})$, but instead we follow the prescription of Myers and Swiatecki [8], i.e. we define an equivalent sharp surface, and add a diffuseness correction term, which is independent on the deformation. We also include a Coulomb exchange term of the Slater approximation type [9].

$$E_{ex}(\bar{r}) = -\frac{3e^2}{4}\left(\frac{3}{\pi}\right)^{1/3}\left[\rho_p(\bar{r})\right]^{4/3} \tag{24}$$

3. PARAMETERIZATION OF THE DENSITY

Since we use the density (and not the potential) as the basic input in our calculations, we must find a reliable way of parameterizing the deformed density. We start by defining a sharp surface, as shown in fig. 1. The nucleus is divided into three regions, defined by the matching points c_1 and c_2. The left (I) and the right (III) regions, describing the fragments under formation, are chosen to be parts of ellipsoids, defined by $\pi_i(r,z) = 0$, where

$$\pi_i(r,z) = -r^2 + A_i^2 - \frac{1}{q_i^2}(z - z_i)^2 \qquad (i = 1,2) \tag{25}$$

The index i denotes the two regions and it is possible to choose different deformations for the two fragments. The neck region (II) is described by $\pi_{II}(r,z) = 0$ with

$$\pi_{II}(r,z) = -r^2 + s^2 + tz^2 + uz^3 + vz^4 + wz^5 + yz^6 \tag{26}$$

Since we want the neck to be located at z = 0, $\pi_{II}(r,z)$ does not contain any term proportional to z, and if only cases with reflexional symmetry are considered also the coefficients u and w are zero. Including the matching points, c_i, this parameterization contains 14 parameters. By requiring that r(z) and its three first derivatives are continuous at the matching points the number of parameters is reduced to 6.

Once the sharp surface is determined, the diffuse density is calculated in accordance with ref. [10]. From the functions $\pi_\nu(r,z)$, defined in eqs. (25) and (26), we define a length

$$l_\nu(r,z) = \frac{\pi_\nu(r,z)}{|\nabla \pi_\nu(r,z)|} \tag{27}$$

The index ν refers to the three regions in fig. 1. The density is then given by

$$\rho_\nu(r,z) = \frac{\rho_0}{1 + e^{l_\nu(r,z)/a_s}} \tag{28}$$

The requirement that r(z) and its three first derivatives are continuous, assures that the density and its two first derivatives become continuous at the limits between the three regions. This is a necessary condition for applying the semiclassical formulas to fourth order. The parameters ρ_0 and a_s are not to be considered as free parameters, since they are determined by minimizing the total energy. In eq. (28) ρ_0 denotes the total density.[1] The individual densities of protons and neutrons are also given by eq. (28) after replacing ρ_0 with ρ_0^p and ρ_0^n respectively, where $\rho_0 = \rho_0^p + \rho_0^n$ and $\rho_0^p/\rho_0^n = Z/N$. The particle number is determined by integrating the density. By requiring a given particle number, one of the remaining deformation parameters can be eliminated. Thus in the most general case we are left with 5 independent parameters. These can of course be chosen in different ways. For practical reasons the following choice seems to be most useful:

1. The total length of the nucleus, $L = z_1 + z_2 + A_1 \cdot q_1 + A_2 \cdot q_2$
2. The radius of the neck, s.
3. The maximal radius of the left fragment, a_1, which is equal to A_1 if $c_1 > z_1$.
4. The maximal radius of the right fragment, a_2, which is equal to A_2 if $c_2 < z_2$.
5. The mass ratio M_L/M_R.

In the present calculations we consider only reflexion symmetric shapes. Consequently we are left with only three independent parameters, namely L, s and the maximal radius of the fragments which we call a.

The above parameterization of the shape is adjusted to describe shapes in the last stage of the fission process, where a neck already has started to be formed. It then allows to describe very compact as well as very elongated shapes, and a considerable variation of the deformation of the fragments can be obtained.

With certain modifications the parameterization can be extended to describe the separated fragments after the scission point. It is, however, not suitable for small deformations, i.e. before the neck is formed. This is directly seen in the calculations, since in this region the matching points are reaching the ends of the fragments and we are left with only the polynomial describing the middle region. The parameterization then

[1] Note that definition (27) leads to a constant surface thickness.

becomes equivalent to the one of ref. [10], except that we have a sixth-order polynomial, allowing for three independent deformation parameters (in the reflexion symmetric case), while ref. [10] uses a fourth-order polynomial, allowing for only two independent deformation parameters.

4. SEMI-CLASSICAL FISSION BARRIERS

It has been shown that when using the Strutinsky smoothed density as input, the functional $\tau[\rho]$ [10] reproduces the Strutinsky averaged energy within a few MeV [11]. We should not expect such an accuracy since we make an independent ansatz for the density. Therefore, before comparing the fission barriers calculated with the semi-classical methods, let us mention a few points. First, the "liquid-drop" barrier height for ^{240}Pu is about 4 MeV, whereas the Strutinsky smoothed barrier obtained from self-consistent H.F. calculations with the Skyrme III potential is about 13 MeV [1]. Furthermore, when defining the density $\rho(\bar{r})$ we assume that the central density ρ_0 and the surface diffuseness, a_s, stay independent of deformation. Moreover, protons and neutrons are assumed to have the same spatial distribution. Self-consistent calculations have shown that these assumptions are not totally satisfactory. Anyhow, we should expect to find a fission barrier which is much higher than the liquid-drop barrier provided our parameterization is reasonable.

The density parameters ρ_0 and a_s have been determined by minimizing the total binding energy at the liquid-drop saddle point. For these calculations we have used the shape parameterization of ref. [10] with c = 1.45 and h = 0. The minimization was done in two cases namely with the semi-classical expansion up to the second order (T_2 term only) and then up to the fourth order (T_4 term included). The respective values obtained in each case were $\rho_0 = 0.139$ fm^{-3}, $a_s = 0.330$ fm and $\rho_0 = 0.140$ fm^{-3}, $a_s = 0.409$ fm. In order to check the dependence on deformation, a minimization was performed for the spherical shape in which case we found practically the same value for ρ_0 and a small shift of a few percent for a_s. In the calculations where this variation was neglected the above values were taken. The central density $\rho_0 = 0.140$ fm^{-3} is in excellent agreement to H.F results of ref. [12]. It should be noted that our density has not the same fall-off as a Fermi distribution since the tail of our distribution extends further outside. However, we have calculated an equivalent Fermi diffuseness, d_0, by fitting a Fermi distribution to our density by requiring same central density ρ_0, half radius and particle number. The Fermi diffusenesses are, for both cases considered above, respectively $d_0 = 0.407$ fm and $d_0 = 0.505$ fm. These values should be compared to $d_0 = 0.398$ fm and $d_0 = 0.420$ fm, respectively, as obtained by Grammaticos et al. [3] when optimizing the surface energy by a semi-classical method. In Fig. 2 we show the semi-classical barriers calculated with the T_2-term only (S.C.2 ; ρ_0=0.139fm) and with both T_2 and T_4 terms (S.C.4; ρ_0=0.140 fm). It should be mentioned that the spin-orbit terms in T_4 have been omitted which may move slightly the barrier S.C.4. The maximum of the Strutinsky smoothed H.F. barrier, marked by a cross, is seen to be just in between. The dashed curve shows the barrier calculated without the T_4 term but for density parameters obtained by considering it. It is seen that once these parameters have been properly optimized (inclusion of T_4) the fourth order term does not influence much on the barrier height except for the largest deformations. It should be underlined that the energies calculated with the T_4 term rise steeply, once one goes away from the optimal values ρ_0 and a_s. This explains the relatively large charge in a_s caused by the inclusion

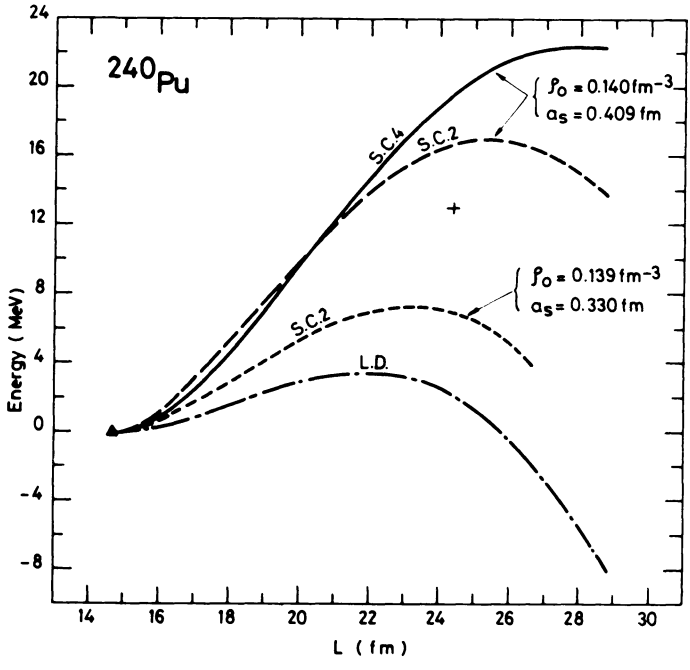

FIG.2. Deformation energy for ^{240}Pu as a function of elongation parameter L. The shape parameterization used for constructing this figure is that of Ref.[10]*. The various curves are discussed in the text.*

of this term. The results shown in fig. 2 are calculated with the {c,h} parameterization of ref. [10] which is very convenient for deformations up to the saddle-point. The parameter h is put equal to zero which corresponds to the liquid-drop valley [10]. The error made by not minimizing with respect to h has been checked for a few cases and found to be negligible.

Note that our calculations in both cases (SC2 and SC4) led to binding energies for the sphere which were less than 5% away from the corresponding Strutinsky smoothed quantity (-1799 MeV as quoted in ref. [1]). Regarding our simplified parameterization of the density distribution and the sensitivity of the binding energy to small changes in the density parameters, this is a fully acceptable result.

5. NUCLEAR DEFORMATIONS INVOLVED IN THE FISSION PROCESS

Although the semi-classical fission barrier heights are too high, they are in relatively good agreement with the smoothed H.F calculations [1] (cf. Fig. 3). It might, therefore, be of interest to investigate the shapes involved in the fission process that the semi-classical calculations predict. For each length, L, of the nucleus the energy has been minimized with respect to the neck radius, s and fragment radius, a. Compared to the

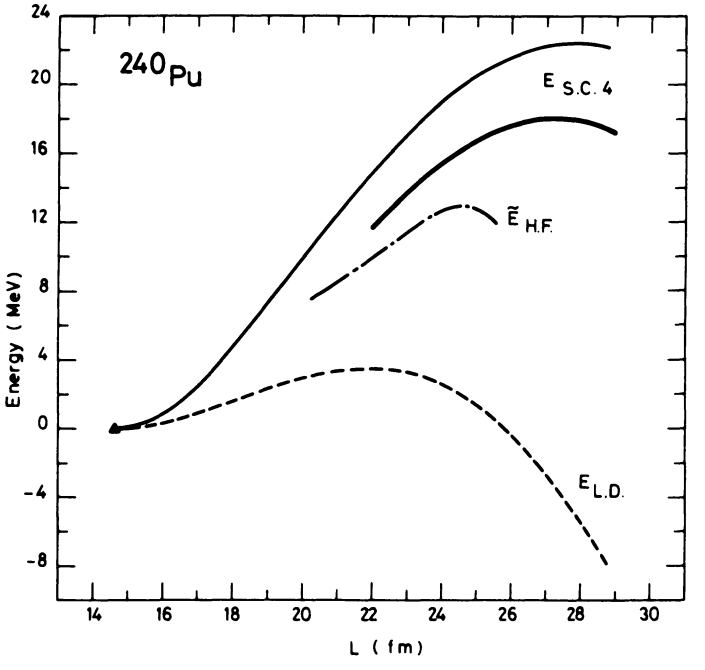

FIG.3. Solid curves show semi-classical deformation energy, calculated with the T_4-term included. The thin line shows the results obtained with the shape parameterization of Ref.[10], *while the thick line is calculated with the parameterization as defined in Section 3. The curve labelled $E_{H.F.}$ shows the Strutinsky-smoothed H.F. energy (taken from Ref.*[1]*) and the curve labelled $E_{L.D.}$ shows the liquid-drop energy, calculated with the shape parameters of Ref.*[10].

{c,h} parameterization the barrier is lowered at the top by about 4.5 MeV (thick solid line in Fig. 3). In Fig. 4 we show the corresponding values of the parameters s and a (thick lines). The neck starts to develop at L $\approx$ 23.5 fm, a length which is almost equal to the sum of the diameters of the resulting fragments shown by the small arrow in Fig. 4. With a further elongation the fragment radius decreases slightly but beyond L $\approx$ 27 fm it starts to increase. We interpret this behaviour as if the system starts to feel the individualities of the fragments under formation and thus tries to prevent too elongated fragments since in this semi-classical frame the sphere is energetically favoured. The neck radius is a smoothly decreasing function of L up to L $\approx$ 29 fm. However, for L > 29 fm the binding energy decreases drastically with decreasing neck radius and there is no longer any static barrier in the s-direction which prevents the fragments to separate, i.e. the "exit-point" is reached. These results are in complete agreement with that of ref. [10] based on the liquid-drop model. The parameter values

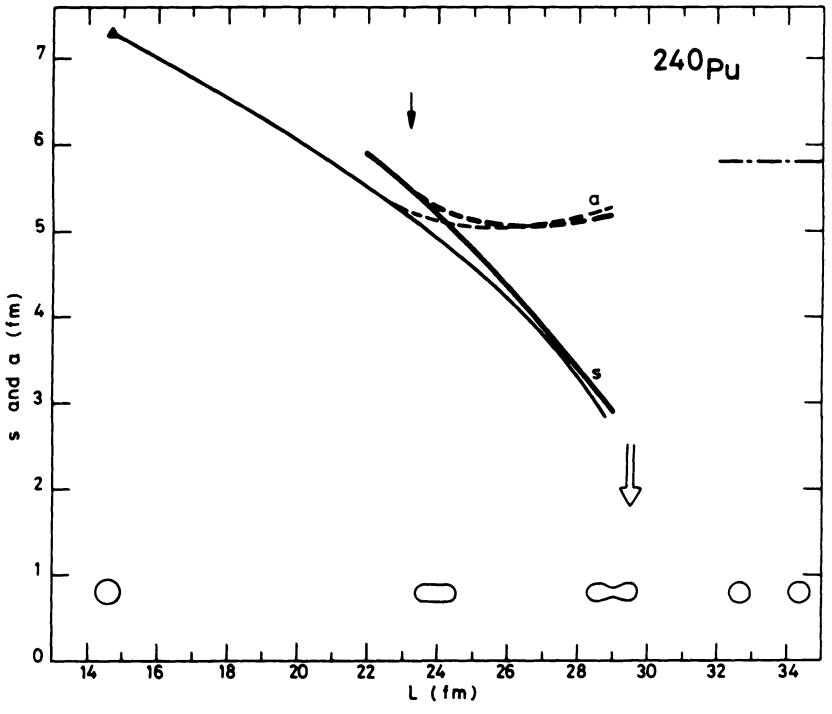

FIG.4. Deformation parameters a and s along the fission path. Thin lines correspond to the shape parameterization of Ref.[10] *and thick lines to that of Section 3. The triangle specifies the spherical shape of* ^{240}Pu *and the open arrow indicates the exit point. The dot-dashed line shows the fragment radius of the two completely separated fragments, which is the limiting value of a for large L. Some typical shapes are shown at the bottom of the figure.*

derived from this reference are also shown in Fig. 4. (Notice that, in this case, a is dependent on s and L and cannot be chosen freely). The results are seen to be rather similar. A somewhat larger difference is obtained on the axis ratio of the fragment as shown in Fig. 5. This axis ratio, q', is defined as $q' = b/a$ where $b = \frac{L}{2} - |z_m|$ with z_m being such that $r(z_m) = a$. (see Fig. 1). In both cases q' decreases strongly with increasing L and goes to 1 near the "exit point". It should be noted that along the fission path the matching points c_i ly outside z_i that is the shapes of the nascent fragments deviate quite much from pure ellipsoidal shapes. Fig. 6 shows some shapes obtained along the fission path.

6. SUMMARY

Our calculations based on the Skyrme-III potential have shown that semi-classical techniques can be applied for describing very deformed nuclei. Regarding the simplifications used in our calculations we find the

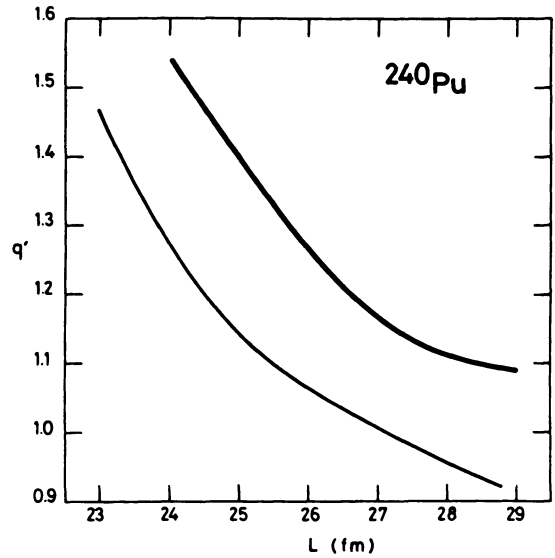

FIG.5. Axis ratio, q', of fragments (for definition, see text). The thin line refers to the parameterization of Ref.[10], *while the thick line refers to that of Section 3.*

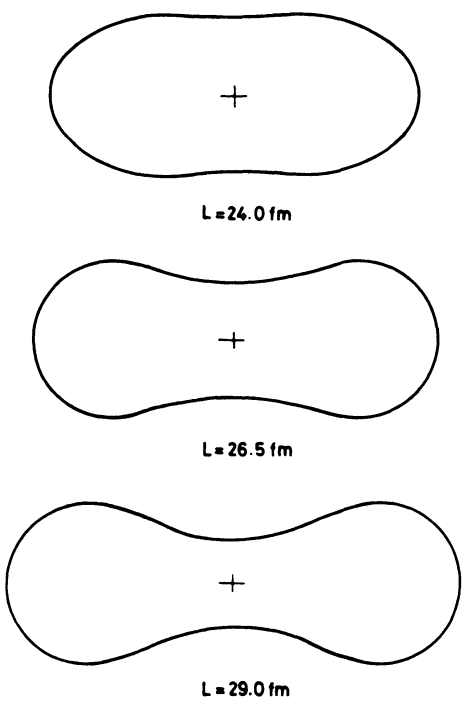

FIG.6. Three shapes along the fission path obtained with the parameterization of Section 3.

agreement with the Strutinsky-smoothed H.F results very encouraging. The too high height for the fission barrier should not be understood as a deficiency of the semi-classical technique but rather as inherent to the Skyrme parameters. As the semi-classical calculations are much less time-consuming than the quantum-mechanical H.F calculations they could easily be used to refit the Skyrme parameters so that they also give reasonable fission barriers. This task would require an improved parameterization of the density or, preferably, a self-consistent calculation of it.

The calculated nuclear shapes along the fission path might be somewhat influenced by the failure of the Skyrme parameters to calculate energies of large deformations. In spite of this uncertainty a strong support is brought to the existence of an "exit point" as obtained in ref. [10]. This "exit-point" is reached for a relatively compact shape of the fissioning system, the neck radius being between 2.5 fm and 3.0 fm.

It should finally be mentioned that it is also possible to treat pair correlations semi-classically. For a preliminary report see ref. [13].

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DISCUSSION

M. ASGHAR: Your calculation clearly brings out the presence of the 'exit point' predicted by Strutinsky almost 15 years ago solely on the basis of his liquid-drop-model research.

But I want to ask Professor Fong whether the shapes of these fragments at the exit point are similar to those assumed in his work a long time ago.

P. FONG *(Chairman)*: Yes, indeed they are. My earlier calculation was based on rather simple assumptions for the sake of convenience. If it closely corresponds to reality, it is perhaps more by luck than judgement.